V.I. Isakov[1], K.I. Erokhina[2], H. Mach[3], M. Sanchez-Vega[3], B. Fogelberg[3]

[1] *Petersburg Nuclear Physics Institute, Russian Academy of Sciences, Gatchina 188300, Russia*

[2] *Physicotechnical Institute, Russian Academy of Sciences, St.Petersburg 194021, Russia*

[3] *Department of Radiation Sciences, Uppsala University, Nyköping S-61162, Sweden*

By the use of the latest experimental data on the spectra of ^{133}Sb and ^{131}Sn and on the analysis of properties of other odd nuclei adjacent to doubly magic closed shells the isospin dependence of a mean spin-orbit potential is defined. Such a dependence received the explanation in the framework of different theoretical approaches.

Recent experimental results [1–3] on nuclei close to ^{132}Sn have lead to the determination of a nearly complete set of neutron and proton single-particle orbitals and to the establishment of some of their important statical and dynamical properties. In particular, the new results [2] for ^{133}Sb include information on energies of proton single particle states above the $Z = 50, N = 82$ shells as well as important knowledge about the decay properties of these states. In view of new data on single particle levels at ^{132}Sn we performed an analysis [4] of the available information on such states in strongly magical nuclides, with special attention to the magnitudes of the spin-orbit splittings and their isospin dependence. This question is important since such a dependence could be one of the factors contributing to significant structural changes in nuclides having an extreme neutron excess.

In [2] the energy of the $3/2^+$ level in ^{133}Sb was measured to be 2.44 MeV. By using this value and also the previous data on the spectrum of single particle excitations in nuclei close to ^{132}Sn (see [1] and [5–8]) the values of spin-orbit splittings of the $2d$ levels both in proton and neutron systems of ^{132}Sn were defined. The splitting was found to be 1.48 MeV for protons and 1.65 MeV for neutrons, i.e. the neutron spin-orbit splitting is somewhat larger for neutrons than for protons. At the same time it was noted in [2] that for nuclei close to ^{208}Pb the situation is the opposite, in any case for the first glance. So, from the spectra of single particle levels in ^{209}Bi and ^{207}Pb it follows that the spin-orbit splitting of the proton $2f$ orbit is equal to 1.93 MeV, while for neutrons it is 1.77 MeV. However it follows from the experiment that the neutron $2f7/2$ state in ^{207}Pb is strongly fragmented. So, the conclusions of the work [2] refer only to the lowest, though the strongest component of this state. Identifying in the spirit of [9], [10] the true single particle energy of the $2f7/2$ state with the weighted average of $7/2^-$ energy levels, the weight being the spectroscopic factors of the (d,t) reaction on ^{208}Pb [11], we obtain the real excitation energy of this state equal to 2.70 MeV (instead of 2.34 MeV). This corresponds to the value of neutron spin-orbit splitting of the $2f$ orbit equal to 2.13 MeV, i.e. as for the $2d$ orbit in ^{132}Sn a little larger than that for protons. The above statement is fortified by the analysis of the $3p$ spin-orbit splitting near ^{208}Pb. Such a splitting is equal to 0.85 MeV for protons (taking into account the fragmentation of $3p1/2$ level) while for neutrons it is 0.90 MeV, i.e. a little larger than for protons.

The systematics of single particle energies in ^{208}Pb and ^{132}Sn available by now is presented in Tables 1 and 2. In composing these tables the energies of the particle and hole states closest to the Fermi-level were determined from the differences of binding energies of the core and the corresponding adjacent odd nuclei: ε(particle)=B(core)–B(core+nucleon) and ε(hole)=B(core–nucleon)–B(core), using the experimental data from [12]. The energies of orbitals more remote from the Fermi-level were defined after that by the addition (subtraction) of the excitation energies [1], [2], [5]–[8], [13], [14] of the corresponding orbitals in the adjacent odd nuclei, accounting for the fragmentation of states, if the corresponding data are available. It really follows from Tables 1 and 2 that the neutron spin-orbit splitting in magical nuclei ^{208}Pb and ^{132}Sn, both having considerable neutron excess as compared to the number of protons, is larger than the corresponding proton splitting by about $\sim 10\%$.

One can see a completely different picture in the $N = Z$ nuclei; see Tables $3 \div 5$. Here the spin-orbit splittings of the $1d$ and $1p$ levels in ^{16}O [15], [16] as well as those of $1f$ and $2p$ levels in ^{40}Ca [17] are practically equal, another time suggesting the concept of isobaric invariance in nuclei. By the present time the experimental data on the structure of single particle states in ^{100}Sn are absent. However the work [18] presents their spectrum obtained from the extrapolation of the properties of nuclei with less neutron deficiency (see Table 5). A conclusion based on these data is that, within the errors the spin-orbit splitting of the $1g$ orbit is also equal for protons and neutrons.

Turning to the theoretical interpretation [4] of data on the spin-orbit splitting we shall first recall that from the point of view of multiparticle theory the average spin-orbit potential has it's origin in the pair spin-orbit interaction between nucleons (with the tensor forces also giving some contribution in the second order perturbation theory). On the level of qualitative arguments it was noted by ref. [19] that due to the symmetry properties one should expect the neutron spin-orbit splitting somewhat larger than that for protons. However, at that time the absence of experimental data did not permit to make a meaningful comparison with measurements. With the presently available data including those obtained by us in [2] we can fill this gap, giving also some quantitative considerations.

The two-body spin-orbit interaction differs from zero only in the states with total spin $S = 1$. Neutron-neutron and proton-proton systems have the total isospin $T = 1$ and thus due to the Pauli principle have odd values of the relative orbital momentum L (really, $L = 1$). At the same time, the neutron-proton system is composed from the $T = 0$ and $T = 1$ states with equal weights, correspondingly having $L = 0$ and $L = 1$. However, the spin-orbit interaction is absent in states with $L = 0$. As a result, the pair spin-orbit np interaction is half as strong as that in pp or nn-systems.

If $U_{\ell s}(n)$ and $U_{\ell s}(p)$ are the values presenting the magnitudes of the mean spin-orbit field for neutrons and protons and $\vartheta(T = 1, S = 1, L = 1)$ is a quantity representing the parameter of the pair spin-orbit interaction in a state with $T = 1, S = 1, L = 1$ then the above discourse leads to

$$U_{\ell s}(n) \sim \vartheta(1,1,1)\left(N + \frac{1}{2}Z\right) \equiv \vartheta \cdot \left(A - \frac{Z}{2}\right) \text{ and}$$

$$U_{\ell s}(p) \sim \vartheta(1,1,1)\left(\frac{N}{2} + Z\right) \equiv \vartheta \cdot \left(A - \frac{N}{2}\right). \quad (1)$$

As the spin-orbit splitting $\Delta_{\ell s}^{(n,p)} \sim U_{\ell s}(n,p)$, the relative difference "ε" of the neutron and proton spin-orbit splittings is given by the expression:

$$\varepsilon = \frac{\Delta_{\ell s}^{(n)} - \Delta_{\ell s}^{(p)}}{(\Delta_{\ell s}^{(n)} + \Delta_{\ell s}^{(p)})/2} = \frac{2}{3}\,\frac{N - Z}{A}. \quad (2)$$

On the other side, if we express the parameter of the spin-orbit mean field in the form

$$U_{\ell s}(\tau_3) = V_{\ell s}\left(1 + \frac{1}{2}\,\beta_{\ell s}\frac{N - Z}{A} \cdot \tau_3\right), \quad (3)$$

where $\tau_3 = -1$ for neutrons and $\tau_3 = +1$ for protons, then

$$\varepsilon = -\beta_{\ell s}\frac{N - Z}{A}, \quad (4)$$

i.e. it follows from the comparison of (2) and (4) that $\beta_{\ell s} = -2/3$.

One can also make the evaluation of the isotopical dependence of spin-orbit interaction in the Hartree approximation starting from the Dirac phenomenology with meson-nucleon interactions [20]. There one obtains (see for example [21]– [28] and the references therein) the Skyrme-type single particle equation for a nucleon having the effective mass m_N^*. Here we concentrate on the isotopical dependence of the spin-orbit potential having the form, see for example [24]– [27]:

$$\hat{U}_{\ell s} = \frac{\lambda_N^2}{2}\,\frac{1}{r}\,\{(\frac{m_N}{m_N^*})^2\frac{d}{dr}[(V_\omega^0 - S_{\sigma,\sigma_0}^0) -$$

$$-(V_\rho^1 - S_{\delta,\sigma,\sigma_0}^1)\cdot\tau_3] - 2k(\frac{m_N}{m_N^*})\frac{d}{dr}V_\rho^1\cdot\tau_3\}\,\hat{\ell}\cdot\hat{\mathbf{s}}. \quad (5)$$

Here $V = V^0 - \tau_3 \cdot V^1$ and $S = S^0 - \tau_3 \cdot S^1$ are the vector and scalar fields due to corresponding mesons, $m_N^* = m_N + \frac{1}{2}(S - V)$, while "$k$" is the ratio of tensor to vector coupling constants of ρ-meson. In [27] the meson-nucleon coupling constants defining the V and S fields were borrowed from the Bonn NN boson exchange potential [29], where σ and σ_0 are scalar mesons imitating the 2π exchange in the NN- systems with T=1 and T=0 correspondingly. At the same time, in some other works (see for example [24]– [26]) the mentioned constants were defined from the description of global nuclear properties, with inclusion of the σ^3 and σ^4 terms in the Lagrangian density (one σ-meson with the same characteristics for T=1 and T=0 channels was used which leads to zero contribution of this meson to S^1 in formula (5), the tensor term was not included in the ρ-meson vertex in the cited works). Taking into account that the radial dependence of the (m_N/m_N^*) is much weaker than that of V and S, which are considered as proportional to the density having the Fermi–function form, one can approximately present formula (5) as

$$\frac{1}{x}\,\frac{df}{dx}\cdot V_{\ell s}\left(1 + \frac{1}{2}\,\beta_{\ell s}\frac{N - Z}{A}\cdot\tau_3\right)\hat{\ell}\cdot\hat{\mathbf{s}}\,;$$

$$f = [1 + exp\,(\frac{x - R}{a})]^{-1}\,. \quad (6)$$

Calculating the V and S magnitudes in the center of nuclei at the values of vector and scalar densities $\rho_v = 0.17$, $\rho_s = 0.16$, $\rho_v^- = 0.17\,(N - Z)/A$, $\rho_s^- = 0.16\ (N - Z)/A$ (all in fm^{-3}) , using the coupling parameters from [27], [29] and taking into account the isotopic dependence of m_N/m_N^*, we obtain $V_{\ell s} = 33.6$ MeV and $\beta_{\ell s} = -0.40$ with "x" in the units of fm. If we use the set of parameters NL2 from [25,26] than we have $V_{\ell s} = 31.3$ MeV, $\beta_{\ell s} = -\,0.43$. At the same time, the set NL1 from [24,26] giving small values of effective masses leads to $V_{\ell s} \approx 50$ MeV and $\beta_{\ell s} \approx -\,1.3$. As the V^1, S^1 magnitudes are proportional to ρ_v^-, ρ_s^- both the formulas (5) and (6) give the spin-orbit splitting equal for protons and neutrons in the $N = Z$ nuclei. It should be mentioned that in all cases the value of $\beta_{\ell s}$ is always negative and is defined mainly or entirely by contribution of a ρ-meson.

We note here that the data on spin-orbit splittings of the $2d$ states in ^{132}Sn as well as on the splittings of $2f$

and $3p$ levels in ^{208}Pb lead to effective values of $\beta_{\ell s}$ equal to -0.55, -0.60 and -0.27 correspondingly.

Let single particle levels be generated by the potential

$$\hat{U}(x,\hat{\sigma},\tau_3) = U_0(\tau_3) f(x) +$$

$$+ \frac{U_{\ell s}(\tau_3)}{x}\frac{df}{dx}\,\hat{\ell}\cdot\hat{\mathbf{s}} + \frac{(1+\tau_3)}{2}\,U_{Coul}\,, \qquad (7)$$

where $U_0(\tau_3) = V_0(1+\frac{1}{2}\beta\frac{N-Z}{A}\cdot\tau_3)$; $U_{\ell s}$ and $f(x,a,R)$ are defined by eq. (3) and (6), $R = r_0A^{1/3}$, while $U_{Coul}(x,R_c,Z)$ presents the potential of the uniformly charged sphere with the charge Z and radii $R_c = r_cA^{1/3}$.

In works [30] – [34] single particle levels were described by using the set of parameters $V_0 = -51.5$ MeV, $r_0 =$ 1.27 fm, $V_{\ell s}$ =33.2 MeV, $a(p)$= 0.67 fm, $a(n)$ = 0.55 fm and $\beta_{\ell s}$ =β =1.39, which on the average described the spectra of single particle states in nuclei from ^{16}O to ^{208}Pb. This set of parameters we call as the "Standard" one. With the appearance of new experimental data on the single-particle levels we performed a new determination of parameters entering formula (7). They were defined by using the Nelder–Mead method [35] through minimization of the root-mean square deviation

$$\delta = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(\varepsilon_k^{\rm theor} - \varepsilon_k^{\rm exp})^2}\,. \qquad (8)$$

The computation demonstrated a very small sensitivity of results to the values of r_c, which was adopted by us the same as before, $r_c = 1.25$ fm. The minimization of δ held for all nuclei presented in Tables $1 \div 5$ with $r_c = 1.25$ fm and different values of r_0 showed that the minimum in all cases corresponds to $r_0 \approx 1.27$ fm which also coincides with the value adopted by us before. The values $r_c = 1.25$ fm and $r_0 = 1.27$ fm were fixed in further calculations.

As was noted above, the optimal relation of proton to neutron spin-orbit splitting corresponds to $\beta_{\ell s} \sim -0.6$. The fourth column of Tables 1 and 2 (variant 1 of calculations) presents the values of theoretical energy levels obtained in the optimization with fixed values of $\beta_{\ell s} = -0.6$, $a_p = 0.67$ fm and $a_n = 0.55$ fm.

The fifth column of Tables $1 \div 2$ (variant 2) presents the results of optimization with two fixed parameters: $a_p = 0.67$ fm and $a_n = 0.55$ fm.

Variant 3 corresponds to optimization at fixed $\beta_{\ell s} =$ -0.6, while variant 4 presents the results with no parameters fixed.

We see that the optimal values of V_0, $V_{\ell s}$ and β (see formula (7)) are very close to the "Standard" ones, with small dispersion from nuclei to nuclei. The magnitudes of the diffusinesses "a" vary more strongly, differing by about $10 \div 15\%$ from their "standard" values. As one can see from the comparisons of the "Stnd" with "Set 1" and "Set 3" with "Set 4" fittings, the contribution of $\beta_{\ell s}$ that defines the isospin dependence of spin-orbit splitting to δ is small. It is more reasonable to define it's value not from minimization of δ, but from experimental and theoretical arguments mentioned above. This conclusion is confirmed by the result of [37], where different fittings gave diverse (in magnitudes and signs) values of parameter defining the linear in $(N-Z)/A$ contribution to the spin-orbit term (energies with maximal values of spectroscopic factors were used as the input ones in these fittings).

The energies of levels in nuclei with $N = Z$ (see Tables 3,4,5) are independent on β and $\beta_{\ell s}$. Here the optimization was performed twice, once with fixed values of $a_n = 0.55$ fm and $a_p = 0.67$ fm with subsequent definition of V and $V_{\ell s}$ (variant 1) and once without fixing some parameters (variant 3).

The results of calculations presented in Tables 1 to 5 refer also to some levels having positive energies, i.e. to unbound, but sub-barrier states. In these cases we present here only the real part of single particle energies having here really very small decay widths.

For comparison of the results obtained by using the empirical potential (7) with those using the microscopical procedure we also performed Hartree-Fock calculations with the SIII interaction (last two columns of Tables 1 to 5). The results of our self-consistent calculations were obtained by considering the contribution of a single-particle part of the center-of-mass energy and taking into account the Coulomb exchange term in the Slater approximation. The SIII(1) data correspond to calculations taking into account all the terms of the energy functional contributing to spin-orbit splitting, while the SIII(2) results were obtained by omitting the spin density terms in the spin-orbit potential. In the last case our results are close to that from the work [36] for ^{208}Pb, ^{132}Sn and ^{100}Sn nuclei. We see that the results obtained in the framework of the Hartree-Fock method also demonstrate that the neutron spin-orbit splittings of the $2d$ orbit in ^{132}Sn as well as of the $2f$ and $3p$ orbits in ^{208}Pb are larger than those for protons and correspond to effective $\beta_{\ell s}$ in the interval of $-0.9 \div -0.6$. One can mention here that the difference between the neutron and proton spin-orbit splittings is reproduced by using the simple parameterization of Skyrme forces. More careful parameterization enables to reproduce [38] the anomalous kink of isotopic shifts in Pb isotopes.

Tables 3 and 4 presented below refer to ^{16}O and ^{40}Ca, which are the spin saturated nuclei. In these cases the spin density terms in practice do not contribute the spin-orbit splitting (the corresponding contributions in these cases are due only to small distinction of radial wave functions of spin-orbit partners). One can see, that the SIII-1 and SIII-2 calculations give here almost similar results.

We conclude that our analysis, based both on experimental data and on different theoretical approaches has defined the isotopic dependence of the nuclear mean field spin-orbit splitting. The splitting becomes larger for neutrons than for protons in nuclei with $N > Z$. Importantly, the theoretical analysis shows that the difference between the neutron and proton splittings becomes saturated which precludes very large differences. The rather modest difference seen in the ^{132}Sn region is already about 25% of the saturation value, showing that the isospin dependence can not introduce major structural changes even in extreme cases of neutron excess.

This work was supported by the Swedish Natural Research Council, the Royal Swedish Academy of Sciences and the Russian Foundation of Fundamental Research (grant No. 96-15-96764). The authors are grateful to B.L. Birbrair for discussion.

[1] P. Hoff et al., Phys. Rev. Lett. **77**, 1020 (1996).

[2] M. Sanchez-Vega, B. Fogelberg, H. Mach, R.B.E. Taylor, A. Lindroth, Phys. Rev. Lett. **80**, 5504 (1998).

[3] N. J. Stone et al., Phys. Rev. Lett. **78**, 820 (1997).

[4] V. I. Isakov, K. I. Erokhina, H. Mach, M. Sanchez- Vega, B. Fogelberg, Preprint PNPI, Gatchina, No 2375 (2000).

[5] T. Bjornstad et al., Nucl.Phys. **A453**, 463 (1986).

[6] Yu.V. Sergeenkov, Yu.L. Khazov, Nucl.Data Sheets **72**, 487 (1994).

[7] Yu.V. Sergeenkov, V.M. Sigalov, Nucl.Data Sheets **49**, 639 (1986).

[8] G.A. Stone, S.H. Faller, W. Walters, Phys.Rev. C **39**, 1963 (1989).

[9] B.L. Cohen, R.H. Fulmer, A.L. McCarthy, P. Mukherjee, Rev.Mod.Phys **35**, 332 (1963).

[10] M. Baranger, Nucl.Phys. **A149**, 225 (1970).

[11] R.A. Moyer, B.L. Cohen, R.C. Diehl, Phys.Rev. C **2**, 1898 (1970).

[12] G. Audi, A.H. Wapstra, Nucl.Phys. **A565**, 1 (1993).

[13] M.J. Martin, Nucl.Data Sheets **63**, 723 (1991).

[14] M.J. Martin, Nucl.Data Sheets **70**, 315 (1993).

[15] D.R. Tilley, H.R. Weller, C.M. Cheves, Nucl.Phys. **A564**, 1 (1993).

[16] F. Ajzenberg-Selove, Nucl.Phys. **A523**, 1 (1991).

[17] P.M. Endt, Nucl.Phys. **A521**, 1 (1990).

[18] H. Grawe, R. Shubart, K.H. Maier, D. Seweryniak, Physica Scripta T**56**, 71 (1995).

[19] A. Bohr, B.R. Mottelson, *Nuclear Structure*, vol.1, New York -Amsterdam, 1969.

[20] J.D. Walecka, Annals of Physics **83**, 491 (1974).

[21] R. Brockmann, Phys.Rev C **18**, 1510 (1978).

[22] J.V. Noble, Nucl.Phys. **A329**, 354 (1979).

[23] B. L. Birbrair, L. N. Savushkin, V. N. Fomenko, Sov.J.Nucl.Phys. **35**, 664 (1982).

[24] P.-G. Reinhardt, M. Rufa, J. Martin, W. Greiner, J. Freidrich, Z.Phys. **A323**, 13 (1986).

[25] Suk-Joon Lee, J. Fink, A.B. Balantekin, M.R. Strayer, A.S. Umar, P.-G. Reinhard, J.A. Maruhn, W. Greiner, Phys.Rev.Lett **57**, 2916 (1986).

[26] W. Koepf, P. Ring, Z.Phys. **A339**, 81 (1991).

[27] B.L. Birbrair, Preprint PNPI, Gatchina, No 2234 (1998).

[28] S. Yoshida, H. Sagawa, Nucl.Phys. **A658**, 3 (1999).

[29] R. Macleidt, K. Holinde, Ch. Elster, Physics Reports **149** No1, 1 (1987).

[30] J. P. Omtvedt et al., Nucl. Phys. A (submitted).

[31] K. I. Erokhina, V. I. Isakov, B. Fogelberg and H. Mach, Preprint PNPI, Gatchina, No 2225 (1998).

[32] K. I. Erokhina, V. I. Isakov, B. Fogelberg and H. Mach, Preprint PNPI, Gatchina, No 2136 (1996).

[33] K. I. Erokhina, V. I. Isakov, Physics of Atomic Nuclei **57**, 198 (1994).

[34] K. I. Erokhina, V. I. Isakov, Physics of Atomic Nuclei **59**, 589 (1996).

[35] J.A. Nelder, R. Mead, Computer J. **7**, 308 (1967).

[36] G.A. Leander, J. Dudek, W. Nazarewicz, J.R. Nix, Ph. Quenteen, Phys.Rev.C **30**, 416 (1984).

[37] H. Koura, M. Yamada, Nucl.Phys. **A671**, 96 (2000).

[38] M.M. Sharma, G. Lalazissis, J. König, P. Ring, Phys.Rev.Lett. **74**, 3744 (1995).

[39] A.-M. Oros et al., in *Proceedings of the 6-th International Spring Seminar on Nuclear Physics: Highlights of Modern Nuclear Structure*, S.Agata, 1998.

Table 1. Single particle states of ^{208}Pb.

$n\ell j$	ε_{exp}	Stnd	Set 1	Set 2	Set 3	Set 4	SIII-1	SIII-2
$n3d_{3/2}$	-1.40	-0.32	-0.02	-0.23	-0.96	-0.99	0.38	0.42
$n2g_{7/2}$	-1.44	-0.79	-0.18	-0.65	-0.89	-1.14	0.01	0.14
$n4s_{1/2}$	-1.90	-0.80	-0.70	-0.74	-1.63	-1.51	-0.08	-0.06
$n1j_{15/2}$	-2.09*	-2.42	-3.05	-2.31	-2.23	-1.55	-1.41	-1.93
$n3d_{5/2}$	-2.37	-1.50	-1.45	-1.40	-2.35	-2.13	-0.39	-0.38
$n1i_{11/2}$	-3.16	-4.24	-3.37	-4.05	-2.71	-3.33	-3.37	-2.77
$n2g_{9/2}$	-3.94	-3.71	-3.82	-3.59	-4.24	-3.88	-2.91	-2.97
$n3p_{1/2}$	-7.37	-7.32	-6.94	-7.17	-7.59	-7.61	-7.21	-7.13
$n2f_{5/2}$	-7.94	-8.42	-7.87	-8.25	-8.17	-8.38	-8.59	-8.44
$n3p_{3/2}$	-8.27	-8.18	-8.03	-8.04	-8.59	-8.43	-8.18	-8.15
$n1i_{13/2}$	-9.00	-9.21	-9.62	-9.08	-8.84	-8.31	-9.73	-10.21
$n2f_{7/2}$	-10.07*	-10.57	-10.57	-10.43	-10.72	-10.46	-11.21	-11.24
$n1h_{9/2}$	-10.78	-12.06	-11.35	-11.87	-10.60	-11.09	-13.16	-12.67
$p3p_{1/2}$	0.17*	0.63	0.43	0.72	0.29	0.47	2.79	2.88
$p3p_{3/2}$	-0.68	-0.45	-0.46	-0.35	-0.58	-0.69	1.99	2.03
$p2f_{5/2}$	-0.97	-0.68	-1.03	-0.60	-1.03	-0.61	0.60	0.74
$p1i_{13/2}$	-2.19	-2.86	-2.37	-2.71	-1.94	-2.78	-1.20	-1.53
$p2f_{7/2}$	-2.90	-3.38	-3.24	-3.26	-3.21	-3.53	-1.64	-1.66
$p1h_{9/2}$	-3.80	-4.60	-5.11	-4.53	-4.71	-4.01	-4.68	-4.24
$p3s_{1/2}$	-8.01	-7.76	-7.86	-7.67	-7.87	-7.87	-7.39	-7.33
$p2d_{3/2}$	-8.36	-8.41	-8.66	-8.32	-8.59	-8.30	-8.64	-8.51
$p1h_{11/2}$	-9.36	-9.33	-8.99	-9.18	-8.60	-9.21	-9.35	-9.65
$p2d_{5/2}$	-10.04*	-10.10	-10.05	-9.98	-9.96	-10.15	-10.29	-10.28
$p1g_{7/2}$	-12.18*	-12.07	-12.45	-11.99	-12.08	-11.58	-13.94	-13.59

The "standard" set of parameters corresponds to $V_0 = -51.50$ MeV, $V_{\ell s} = 33.2$ MeV, $\beta = \beta_{\ell s} = +1.39$, $a_p = 0.67$ fm, $a_n = 0.55$ fm and $\delta = 0.604$ MeV.

Set "1" corresponds to $V_0 = -51.39$ MeV, $V_{\ell s} = 33.1$ MeV, $\beta = 1.43$ with $\beta_{\ell s} = -0.6$, $a_p = 0.67$ fm, $a_n = 0.55$ fm fixed; $\delta = 0.654$ MeV.

Set "2" corresponds to $V_0 = -51.34$ MeV, $V_{\ell s} = 33.1$ MeV, $\beta = 1.40$, $\beta_{\ell s} = 1.26$ with $a_p = 0.67$ fm, $a_n = 0.55$ fm fixed; $\delta = 0.593$ MeV.

Set "3" corresponds to $V_0 = -51.99$ MeV, $V_{\ell s} = 32.7$ MeV, $\beta = 1.36$, $a_p = 0.73$ fm, $a_n = 0.72$ fm with $\delta = 0.369$ MeV; $\beta_{\ell s} = -0.6$ is fixed.

Set "4" corresponds to $V_0 = -51.93$ MeV, $V_{\ell s} = 35.2$ MeV, $\beta = 1.38$, $\beta_{\ell s} = 1.76$, $a_p = 0.73$ fm, $a_n = 0.72$ fm; $\delta = 0.366$ MeV.

Here and below experimental single particle energies marked by asterisks were obtained using the averaging over the spectroscopic factors.

Table 2. Single particle states of ^{132}Sn.

$n\ell j$	ε_{exp}	Stnd	Set 1	Set 2	Set 3	Set 4	SIII-1	SIII-2
$n2f5/2$	-0.58	0.36	0.73	0.46	0.22	-0.01	0.67	0.79
$n3p1/2$	(-0.92)	-0.13	-0.48	-0.09	-0.55	-0.61	0.16	0.20
$n1h9/2$	-1.02	-1.61	-0.84	-1.38	-0.47	-0.97	-0.72	-0.02
$n3p3/2$	-1.73	-0.78	-0.88	-0.77	-1.42	-1.32	-0.16	-0.14
$n2f7/2$	-2.58	-2.18	-2.55	-2.21	-2.84	-2.52	-1.67	-1.71
$n2d3/2$	-7.31	-7.74	-7.45	-7.62	-7.63	-7.77	-8.42	-8.26
$n1h11/2$	-7.55	-7.11	-7.96	-7.23	-7.33	-6.60	-7.69	-8.23
$n3s1/2$	-7.64	-7.68	-7.73	-7.64	-8.03	-7.93	-8.26	-8.21
$n2d5/2$	-8.96	-9.66	-9.94	-9.66	-9.98	-9.69	-10.71	-10.71
$n1g7/2$	-9.74	-10.56	-10.04	-10.39	-9.51	-9.81	-11.92	-11.32
$p3s1/2$	(-6.83)	-6.84	-6.87	-6.80	-6.64	-6.70	-4.97	-4.90
p1h11/2	-6.84	-7.32	-6.66	-7.46	-6.77	-7.48	-5.64	-6.01
$p2d3/2$	-7.19	-6.86	-7.20	-6.74	-7.07	-6.72	-5.93	-5.77
$p2d5/2$	-8.67	-9.36	-9.20	-9.37	-9.04	-9.30	-7.88	-7.88
$p1g7/2$	-9.63	-9.84	-10.41	-9.66	-10.60	-9.81	-10.08	-9.56
$p1g9/2$	-15.71	-14.91	-14.46	-15.00	-14.57	-15.02	-15.03	-15.36
$p2p1/2$	-16.07	-16.01	-16.22	-15.92	-16.14	-15.91	-16.68	-16.55

Stnd: $\delta = 0.589$ MeV.

Set 1: $V_0 = -51.56$ MeV, $V_{\ell s} = 33.3$ MeV, $\beta = 1.39$, $\delta = 0.638$ MeV.

Set 2: $V_0 = -51.44$ MeV, $V_{\ell s} = 34.8$ MeV, $\beta = 1.39$, $\beta_{\ell s} = 1.35$, $\delta = 0.575$ MeV.

Set 3: $V_0 = -51.55$ MeV, $V_{\ell s} = 32.4$ MeV, $\beta = 1.31$, $a_p = 0.63$ fm, $a_n = 0.66$ fm, $\delta = 0.546$ MeV.

Set 4: $V_0 = -51.56$ MeV, $V_{\ell s} = 34.1$ MeV, $\beta = 1.34$, $\beta_{\ell s} = 1.33$, $a_p = 0.65$ fm, $a_n = 0.66$ fm, $\delta = 0.478$ MeV.

Note that some theoretical works [39] postulate that the neutron 1i13/2 state at ^{132}Sn is only 1.9 MeV above the n2f7/2 level. Our calculations unequivocally demonstrate, that this state lies considerably higher, with it's energy equal to +0.55, +1.59 and +1.02 MeV correspondingly for the "Stnd", SIII-1 and SIII-2 variants.

Table 3. Single particle levels of ^{16}O.

$n\ell j$	ε_{exp}	Stnd	Set 1	Set 3	SIII-1	SIII-2
$n1d3/2$	(0.94)	0.89	0.18	0.20	0.66	0.67
$n2s1/2$	-3.27	-3.59	-3.89	-3.31	-2.88	-2.87
$n1d5/2$	-4.14	-6.97	-6.85	-6.41	-6.87	-6.89
$n1p1/2$	-15.67	-15.06	-16.05	-16.33	-14.58	-14.56
$n1p3/2$	(-21.84)	-19.98	-20.25	-20.10	-20.58	-20.59
$p1d3/2$	(4.40)	3.76	2.92	3.48	3.55	3.56
$p2s1/2$	-0.11	-0.89	-1.14	0.22	0.03	0.03
$p1d5/2$	-0.60	-2.76	-2.67	-2.97	-3.57	-3.59
$p1p1/2$	-12.13	-9.95	-10.87	-12.60	-11.17	-11.15
$p1p3/2$	(-18.45)	-14.66	-14.90	-16.40	-17.07	-17.08

Set 1: $V_0 = -52.21$ MeV, $V_{\ell s} = 28.6$ MeV; $a_p = 0.67$ fm, $a_n = 0.55$ fm are fixed.

Set 3: $V_0 = -51.40$ MeV, $V_{\ell s} = 25.7$ MeV, $a_p = 0.45$ fm, $a_n = 0.50$ fm.

Table 4. Single particle states of ^{40}Ca.

$n\ell j$	ε_{exp}	Stnd	Set 1	Set 3	SIII-1	SIII-2
$n1f5/2$	-3.48	-2.57	-3.91	-3.54	-1.49	-1.48
$n2p1/2$	-4.42	-3.35	-4.08	-4.69	-2.20	-2.23
$n2p3/2$	-6.42	-5.71	-6.08	-6.57	-4.09	-4.05
$n1f7/2$	-8.36	-10.43	-10.44	-9.72	-9.92	-9.94
$n1d3/2$	-15.64	-16.21	-17.40	-16.43	-15.53	-15.54
$n2s1/2$	-18.11	-16.51	-17.17	-17.00	-15.94	-15.92
$n1d5/2$	-21.64*	-21.08	-21.44	-20.52	-21.90	-21.90
$p1f5/2$	3.86	4.92	3.79	3.41	4.90	4.91
$p2p1/2$	2.64	2.62	2.11	2.07	3.66	3.64
$p2p3/2$	0.63	0.89	0.60	0.45	2.23	2.26
$p1f7/2$	-1.09	-2.19	-2.18	-2.85	-3.04	-3.06
$p1d3/2$	-8.33	-7.11	-8.25	-9.01	-8.52	-8.53
$p2s1/2$	-10.85	-8.18	-8.78	-9.30	-8.77	-8.75
$p1d5/2$	-14.33*	-12.05	-12.36	-13.19	-14.74	-14.75

Set 1: $V_0 = -52.39$ MeV, $V_{\ell s} = 27.9$ MeV; $a_p = 0.67$ fm and $a_n = 0.55$ fm are fixed.

Set 3: $V_0 = -52.95$ MeV, $V_{\ell s} = 28.2$ MeV, $a_p = 0.63$ fm, $a_n = 0.68$ fm.

Table 5. Single particle states of ^{100}Sn.

$n\ell j$	ε_{sys}	Stnd	Set 1	Set 3	SIII-1	SIII-2
$n1h11/2$	-8.6(5)	-8.66	-9.01	-8.72	-6.35	-6.87
$n2d3/2$	-9.2(5)	-8.90	-9.24	-8.70	-7.84	-7.66
$n3s1/2$	-9.3(5)	-9.16	-9.53	-9.13	-7.58	-7.52
$n1g7/2$	-10.93(20)	-11.64	-12.02	-11.23	-10.33	-9.63
$n2d5/2$	-11.13(20)	-11.62	-11.97	-11.59	-10.07	-10.10
$n1g9/2$	-17.93(20)	-17.23	-17.61	-17.21	-16.54	-17.00
$n2p1/2$	-18.38(20)	-19.14	-19.53	-18.93	-19.08	-18.93
$p1g7/2$	3.90(15)	3.88	3.54	2.70	3.38	4.04
$p2d5/2$	3.00(80)	2.74	2.45	2.64	3.70	3.69
$p1g9/2$	-2.92(20)	-2.01	-2.36	-3.66	-2.74	-3.16
$p2p1/2$	-3.53(20)	-3.48	-3.84	-3.94	-4.80	-4.65
$p2p3/2$	-6.38	-4.95	-5.31	-5.55	-6.22	-6.18
$p1f5/2$	-8.71	-5.54	-5.92	-7.60	-8.43	-7.89

Set 1: $V_0 = -51.97$ MeV, $V_{\ell s} = 33.5$ MeV; $a_p = 0.67$ fm and $a_n = 0.55$ fm are fixed.

Set 3: $V_0 = -51.40$ MeV, $V_{\ell s} = 35.6$ MeV, $a_p = 0.52$ fm, $a_n = 0.56$ fm.